AMLS for an unsymmetric eigenproblem governing free vibrations of fluid-solid structures

Heinrich Voss
voss@tuhh.de

Hamburg University of Technology
Institute of Numerical Simulation

Joint work with Markus Stammberger

TUHH
Technische Universität Hamburg-Harburg

Automated Multi-Level Substructring

AMLS was introduced by Bennighof (1998) and was applied to huge problems of frequency response analysis.

Automated Multi-Level Substructring

AMLS was introduced by Bennighof (1998) and was applied to huge problems of frequency response analysis.

The large finite element model is recursively divided into very many substructures on several levels based on the sparsity structure of the system matrices.

Automated Multi-Level Substructring

AMLS was introduced by Bennighof (1998) and was applied to huge problems of frequency response analysis.

The large finite element model is recursively divided into very many substructures on several levels based on the sparsity structure of the system matrices.

Assuming that the interior degrees of freedom of substructures depend quasistatically on the interface degrees of freedom, and modeling the deviation from quasistatic dependence in terms of a small number of selected substructure eigenmodes the size of the finite element model is reduced substantially yet yielding satisfactory accuracy over a wide frequency range of interest.

Automated Multi-Level Substructring

AMLS was introduced by Bennighof (1998) and was applied to huge problems of frequency response analysis.

The large finite element model is recursively divided into very many substructures on several levels based on the sparsity structure of the system matrices.

Assuming that the interior degrees of freedom of substructures depend quasistatically on the interface degrees of freedom, and modeling the deviation from quasistatic dependence in terms of a small number of selected substructure eigenmodes the size of the finite element model is reduced substantially yet yielding satisfactory accuracy over a wide frequency range of interest.

Recent studies in vibro-acoustic analysis of passenger car bodies where very large FE models with more than six million degrees of freedom appear and several hundreds of eigenfrequencies and eigenmodes are needed have shown that AMLS is considerably faster than Lanczos type approaches.

Outline

1. AMLS for linear eigenproblems
2. Fluid-solid vibrations
3. Structure preserving AMLS for fluid-solid structures
4. AMLS for equivalent symmetric eigenproblems

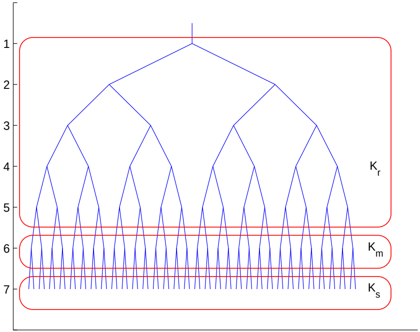
Outline

1. AMLS for linear eigenproblems
2. Fluid-solid vibrations
3. Structure preserving AMLS for fluid-solid structures
4. AMLS for equivalent symmetric eigenproblems

AMLS - Algorithm ($Kx = \lambda Mx$)

Reorder System (using Graph Partitioner):

$$\begin{pmatrix} K_s & K_{sm} & K_{sr} \\ K_{sm}^T & K_m & K_{mr} \\ K_{sr}^T & K_{mr}^T & K_r \end{pmatrix} \quad \text{with} \quad K_s = \begin{pmatrix} K_{s_1} & & \\ & \ddots & \\ & & K_{s_n} \end{pmatrix}$$



AMLS - Algorithm ct.

Congruence transformation with

$$U = \begin{pmatrix} I & -K_s^{-1}K_{sm} & -K_s^{-1}K_{sr} \\ O & I & O \\ O & O & I \end{pmatrix}$$

AMLS - Algorithm ct.

Congruence transformation with

$$U = \begin{pmatrix} I & -K_s^{-1}K_{sm} & -K_s^{-1}K_{sr} \\ O & I & O \\ O & O & I \end{pmatrix}$$

yields

$$\begin{pmatrix} K_s & 0 & 0 \\ 0 & \hat{K}_m & \hat{K}_{mr} \\ 0 & \hat{K}_{mr}^T & \hat{K}_r \end{pmatrix}, \quad \begin{pmatrix} M_s & \hat{M}_{sm} & \hat{M}_{sr} \\ \hat{M}_{sm}^T & \hat{M}_m & \hat{M}_{mr} \\ \hat{M}_{sr}^T & \hat{M}_{mr}^T & \hat{M}_r \end{pmatrix}$$

AMLS - Algorithm ct.

Congruence transformation with

$$U = \begin{pmatrix} I & -K_s^{-1}K_{sm} & -K_s^{-1}K_{sr} \\ O & I & O \\ O & O & I \end{pmatrix}$$

yields

$$\begin{pmatrix} K_s & 0 & 0 \\ 0 & \hat{K}_m & \hat{K}_{mr} \\ 0 & \hat{K}_{mr}^T & \hat{K}_r \end{pmatrix}, \quad \begin{pmatrix} M_s & \hat{M}_{sm} & \hat{M}_{sr} \\ \hat{M}_{sm}^T & \hat{M}_m & \hat{M}_{mr} \\ \hat{M}_{sr}^T & \hat{M}_{mr}^T & \hat{M}_r \end{pmatrix}$$

Notice, that K_s is block-diagonal, and determining $K_s^{-1}K_{sr}$ means that a large number of linear system of small dimension has to be solved. Moreover, the conguence transformation consists of blockmatrix multiplications for blocks of small dimension.

AMLS - Algorithm ct.

Solving of substructure EVPs

$$K_s\Phi_s = M_s\Phi_s\Omega_s, \quad \Phi_s^T M_s\Phi_s = I$$

and projecting on a subset of Φ_s (usually corresponding to eigenvalues not exceeding a cut-off frequency) yields

$$\begin{pmatrix} \tilde{\Omega}_s & 0 & 0 \\ 0 & \hat{K}_m & \hat{K}_{mr} \\ 0 & \hat{K}_{mr}^T & \hat{K}_r \end{pmatrix}, \quad \begin{pmatrix} I_s & \tilde{\hat{M}}_{sm} & \tilde{\hat{M}}_{sr} \\ \tilde{\hat{M}}_{sm}^T & \hat{M}_m & \hat{M}_{mr} \\ \tilde{\hat{M}}_{sr}^T & \hat{M}_{mr}^T & \hat{M}_r \end{pmatrix}$$

AMLS - Algorithm ct.

Solving of substructure EVPs

$$K_s\Phi_s = M_s\Phi_s\Omega_s, \quad \Phi_s^T M_s\Phi_s = I$$

and projecting on a subset of Φ_s (usually corresponding to eigenvalues not exceeding a cut-off frequency) yields

$$\begin{pmatrix} \tilde{\Omega}_s & 0 & 0 \\ 0 & \hat{K}_m & \hat{K}_{mr} \\ 0 & \hat{K}_{mr}^T & \hat{K}_r \end{pmatrix}, \quad \begin{pmatrix} I_s & \tilde{\hat{M}}_{sm} & \tilde{\hat{M}}_{sr} \\ \tilde{\hat{M}}_{sm}^T & \hat{M}_m & \hat{M}_{mr} \\ \tilde{\hat{M}}_{sr}^T & \hat{M}_{mr}^T & \hat{M}_r \end{pmatrix}$$

This first step of AMLS was introduced already by Hurty (1965) and by Craig and Bampton (1968), and it is called Component Mode Synthesis (CMS).

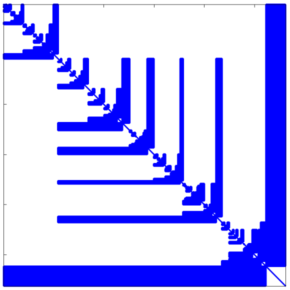
AMLS - Algorithm ct.

Treating coarser levels one after the other in the same way one gets a projected eigenvalue problem of significantly lower dimension

$$K_c x = \lambda M_c x$$

with K_c spd and diagonal and M_c spd in generalized arrowhead structure.

Massmatrix of AMLS



An a priori bound for CMS

After decoupling the substructures on the highest level und introducing modal coordinates the EVP obtains the form

$$\begin{pmatrix} \Omega_1 & & \\ & \Omega_2 & \\ & & \tilde{K}_m \end{pmatrix} \begin{pmatrix} x_{s1} \\ x_{s2} \\ x_m \end{pmatrix} = \lambda \begin{pmatrix} I & & \tilde{M}_{sm1} \\ & I & \tilde{M}_{sm2} \\ \tilde{M}_{sm1}^T & \tilde{M}_{sm2}^T & \tilde{M}_m \end{pmatrix} \begin{pmatrix} x_{s1} \\ x_{s2} \\ x_m \end{pmatrix}$$

where Ω_1 contains the eigenvalues of substructure EVPs which are discarded and Ω_2 the ones that are kept.

An a priori bound for CMS

After decoupling the substructures on the highest level und introducing modal coordinates the EVP obtains the form

$$\begin{pmatrix} \Omega_1 & & \\ & \Omega_2 & \\ & & \tilde{K}_m \end{pmatrix} \begin{pmatrix} x_{s1} \\ x_{s2} \\ x_m \end{pmatrix} = \lambda \begin{pmatrix} I & & \tilde{M}_{sm1} \\ & I & \tilde{M}_{sm2} \\ \tilde{M}_{sm1}^T & \tilde{M}_{sm2}^T & \tilde{M}_m \end{pmatrix} \begin{pmatrix} x_{s1} \\ x_{s2} \\ x_m \end{pmatrix}$$

where Ω_1 contains the eigenvalues of substructure EVPs which are discarded and Ω_2 the ones that are kept.

Eliminating x_{s1} one gets the rational eigenproblem

$$\left[-\begin{pmatrix} \Omega_2 & \\ & \tilde{K}_M \end{pmatrix} + \lambda \begin{pmatrix} I & \tilde{M}_{sm2} \\ \tilde{M}_{sm2}^T & \tilde{M}_M \end{pmatrix} + \lambda^2 \begin{pmatrix} O \\ \tilde{M}_{sm1}^T \end{pmatrix} (\Omega_1 - \lambda I)^{-1} \left(O, \tilde{M}_{sm1} \right) \right] \begin{pmatrix} x_{s2} \\ x_m \end{pmatrix} = 0$$

which has the same eigenvalues as the original EVP (except the ones which are diagonal elements od Ω_1).

An a priori bound for CMS ct.

Let

$$\underline{\omega} := \min \operatorname{diag} \Omega_1$$

the smallest eigenvalue neglected in the CMS method (which can be replaced by the cut off threshold).

An a priori bound for CMS ct.

Let

$$\underline{\omega} := \min \operatorname{diag} \Omega_1$$

the smallest eigenvalue neglected in the CMS method (which can be replaced by the cut off threshold).

Let $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ denote the eigenvalues of $Kx = \lambda Mx$ ordered by magnitude, and let $r \in \mathbb{N}$ such that $\lambda_r < \underline{\omega} \leq \lambda_{r+1}$. Then $\lambda_1, \dots, \lambda_m \in J$ are the eigenvalues of the nonlinear eigenproblem $T(\lambda)x = 0$ in $J := (0, \underline{\omega})$ which can be characterized as minmax values of a Rayleigh functional.

An a priori bound for CMS ct.

Let

$$\underline{\omega} := \min \operatorname{diag} \Omega_1$$

the smallest eigenvalue neglected in the CMS method (which can be replaced by the cut off threshold).

Let $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$ denote the eigenvalues of $Kx = \lambda Mx$ ordered by magnitude, and let $r \in \mathbb{N}$ such that $\lambda_r < \underline{\omega} \le \lambda_{r+1}$. Then $\lambda_1, \dots, \lambda_m \in J$ are the eigenvalues of the nonlinear eigenproblem $T(\lambda)x = 0$ in $J := (0, \underline{\omega})$ which can be characterized as minmax values of a Rayleigh functional.

Comparing the Rayleigh functional p to the Rayleigh quotient of the reduced problem on appropriate subspaces of $\mathbb{R}^r$ we arrive at the following a priori bound for the relative errors of the CMS approximations $\tilde{\lambda}_j$ to λ_j.

$$0 \le \frac{\tilde{\lambda}_j - \lambda_j}{\lambda_j} \le \frac{\lambda_j}{\underline{\omega} - \lambda_j} \le \frac{\tilde{\lambda}_j}{\underline{\omega} - \tilde{\lambda}_j}, \quad j = 1, \dots, r.$$

A priori bound for AMLS

Let K, M and λ_j, $j = 1, \dots, n$ be given as before. Let the graph of $|K| + |M|$ be substructured with p levels, and denote by $\tilde\lambda_1^{(\nu)} \le \tilde\lambda_2^{(\nu)} \le \dots$ the eigenvalues obtained by AMLS with cut-off threshold ω_ν on level ν.
If $r \in \mathbb{N}$ such that

$$\lambda_r < \min_{\nu=0,\dots,p} \omega_\nu \le \lambda_{r+1}$$

then it holds that

$$\frac{\tilde\lambda_j - \lambda_j}{\lambda_j} \le \prod_{\nu=0}^{p} \left(1 + \frac{\lambda_j^{(\nu)}}{\omega_\nu - \lambda_j^{(\nu)}}\right) - 1, \quad j = 1, \dots, r.$$

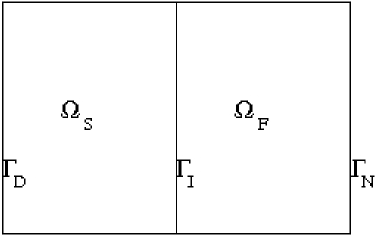
Outline

1. AMLS for linear eigenproblems
2. Fluid-solid vibrations
3. Structure preserving AMLS for fluid-solid structures
4. AMLS for equivalent symmetric eigenproblems

Problem definition

Vibrations of fluid-solid structures can be modelled in terms of solid displacement and fluid pressure and one obtains the classical form of an eigenproblem

$$
\begin{aligned}
\text{div}\,[\sigma(u)] + \omega^2 \rho_s u &= 0 \text{ in } \Omega_s,\\
\Delta p + \frac{\omega^2}{c^2} p &= 0 \text{ in } \Omega_f,\\
\sigma(u)\cdot n - pn &= 0 \text{ on } \Gamma_I,\\
\nabla p \cdot n + \omega^2 \rho_f u \cdot n &= 0 \text{ on } \Gamma_I,\\
u &= 0 \text{ on } \Gamma_D,\\
\nabla p \cdot n &= 0 \text{ on } \Gamma_N,
\end{aligned}
$$



where

- u: solid displacement
- p: fluid pressure
- $\lambda = \omega^2$: eigenparameter
- $\sigma(u)$: linearized stress tensor
- ρ_s, ρ_f: densities of solid and fluid

Variational formulation

Find λ and $(u, p) \in H := (H_0^1(\Omega_s))^3 \times H_1(\Omega_f)$, $(u, p) \neq 0$ such that

$$\int_{\Omega_s} \sigma(u) : \varepsilon(v)\, dx + \int_{\Omega_f} \frac{1}{\rho_f} \nabla p \cdot \nabla q\, dx + \int_{\Gamma_I} p\, n \cdot v\, ds$$

$$= \lambda \left(\int_{\Omega_s} \rho_s u v\, dx + \int_{\Omega_f} \frac{1}{\rho_f c^2} p q\, dx - \int_{\Gamma_I} q\, u \cdot n\, ds \right)$$

for every $(v, q) \in H$.

Variational formulation

Find λ and $(u, p) \in H := (H_0^1(\Omega_s))^3 \times H_1(\Omega_f)$, $(u, p) \neq 0$ such that

$$\int_{\Omega_s} \sigma(u) : \varepsilon(v)\, dx + \int_{\Omega_f} \frac{1}{\rho_f} \nabla p \cdot \nabla q\, dx + \int_{\Gamma_I} p\, n \cdot v\, ds$$

$$= \lambda \left(\int_{\Omega_s} \rho_s uv\, dx + \int_{\Omega_f} \frac{1}{\rho_f c^2} pq\, dx - \int_{\Gamma_I} q\, u \cdot n\, ds \right)$$

for every $(v, q) \in H$.

Properties:

- There exists a countable set of eigenvalues the only cluster point of which is ∞
- All eigenvalues are real and non-negative
- Eigenvalues can be characterized as minmax values of a Rayleigh functional (Stammberger, V. 2009)

Discretization

Discretization by finite elements yields an unsymmetric matrix eigenproblem

$$Kx := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} =: \lambda Mx,$$

Discretization

Discretization by finite elements yields an unsymmetric matrix eigenproblem

$$Kx := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} =: \lambda Mx,$$

where

- K_s, $M_s \in \mathbb{R}^{s \times s}$ are symmetric positive definite stiffness and mass matrices of the solid,
- K_f, $M_f \in \mathbb{R}^{f \times f}$ are symmetric stiffness and mass matrices of the fluid, where K_f is positive semidefinite and M_f positive definite,
- $C \in \mathbb{R}^{s \times f}$ is due to the coupling effects between fluid and solid,
- $x_s \in \mathbb{R}^s$ is the solid displacement vector, and
- $x_f \in \mathbb{R}^f$ the fluid pressure vector.

Usual approach in automotive industry

Solve the uncoupled eigenvalue problem

$$K_u x := \begin{pmatrix} K_s & 0 \\ 0 & K_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ 0 & M_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} =: \lambda M_u x,$$

and project the original problem to diag$\{\Phi_s, \Phi_f\}$, where the columns of Φ_s and Φ_f are eigenmodes corresponding to eigenvalues not exceeding a cut-off frequency.

Usual approach in automotive industry

Solve the uncoupled eigenvalue problem

$$K_u x := \begin{pmatrix} K_s & 0 \\ 0 & K_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ 0 & M_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} =: \lambda M_u x,$$

and project the original problem to diag$\{\Phi_s, \Phi_f\}$, where the columns of Φ_s and Φ_f are eigenmodes corresponding to eigenvalues not exceeding a cut-off frequency.

Eigenpairs of the projected problem are sufficient approximations to exact eigenpairs, if the coupling is weak (fluid is air, structure is metal, e.g.)

Usual approach in automotive industry

Solve the uncoupled eigenvalue problem

$$K_u x := \begin{pmatrix} K_s & 0 \\ 0 & K_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ 0 & M_f \end{pmatrix} \begin{pmatrix} v_s \\ v_f \end{pmatrix} =: \lambda M_u x,$$

and project the original problem to $\text{diag}\{\Phi_s, \Phi_f\}$, where the columns of Φ_s and Φ_f are eigenmodes corresponding to eigenvalues not exceeding a cut-off frequency.

Eigenpairs of the projected problem are sufficient approximations to exact eigenpairs, if the coupling is weak (fluid is air, structure is metal, e.g.)

For strongly coupled problems some eigenpairs are inexact.

Example: Conca et al. 1995

Determine acoustic eigenfrequencies of a (very long) cavity containing a tube bundle.

Example: Conca et al. 1995

Determine acoustic eigenfrequencies of a (very long) cavity containing a tube bundle. Tubes are

- immersed in an inviscid compressible fluid

Example: Conca et al. 1995

Determine acoustic eigenfrequencies of a (very long) cavity containing a tube bundle. Tubes are

- immersed in an inviscid compressible fluid
- rigid, assembled in parallel inside the fluid,

Example: Conca et al. 1995

Determine acoustic eigenfrequencies of a (very long) cavity containing a tube bundle. Tubes are

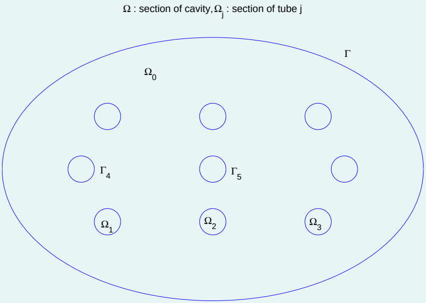
- immersed in an inviscid compressible fluid
- rigid, assembled in parallel inside the fluid,
- elastically mounted such that they can vibrate transversally, but can not move in the direction perpendicular to their sections.

Example: Conca et al. 1995

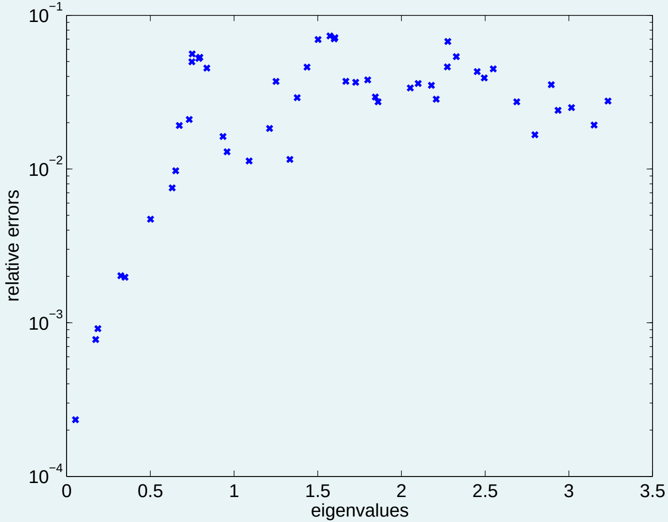
Determine acoustic eigenfrequencies of a (very long) cavity containing a tube bundle. Tubes are

- immersed in an inviscid compressible fluid
- rigid, assembled in parallel inside the fluid,
- elastically mounted such that they can vibrate transversally, but can not move in the direction perpendicular to their sections.

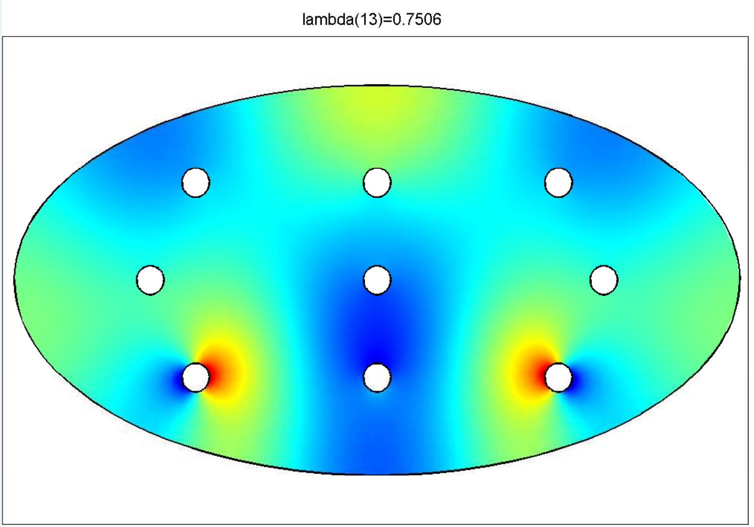
Due to these assumptions, three-dimensional effects are neglected, and the problem is studied in any transversal section of the cavity.



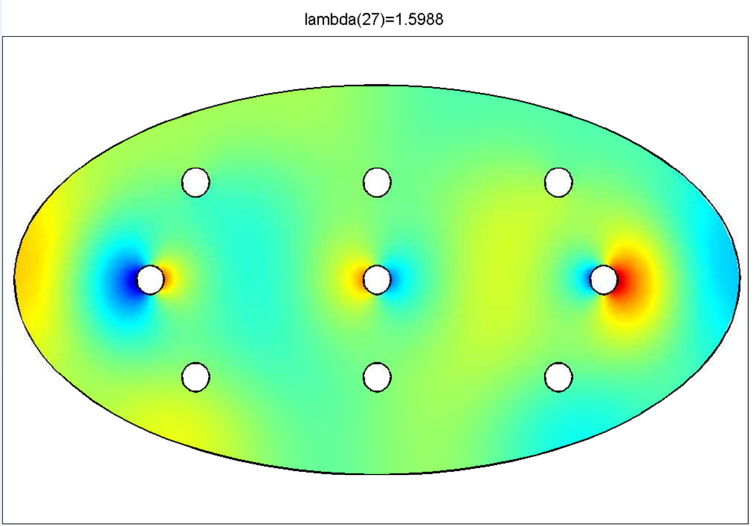
Finite element model: 143081 DoFs



Typical eigenvector



Typical eigenvector



Outline

Structure preserving AMLS

Substructure the joint graph of

$$K := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \quad \text{and} \quad M := \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix}$$

substructured recursively on several levels, where the coupling matrix is incorporated into the substructuring process. Hence, any substructure may consist solely of degrees of freedom from the fluid or from the solid, or caused by the coupling matrix it may contain degrees of freedom of either type.

Structure preserving AMLS

Substructure the joint graph of

$$K := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \quad \text{and} \quad M := \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix}$$

substructured recursively on several levels, where the coupling matrix is incorporated into the substructuring process. Hence, any substructure may consist solely of degrees of freedom from the fluid or from the solid, or caused by the coupling matrix it may contain degrees of freedom of either type.

The crucial point is to modify the AMLS algorithm such that the structure is preserved for the reduced problem, and all eigenvalues are still real.

Structure preserving AMLS

Substructure the joint graph of

$$K := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \quad \text{and} \quad M := \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix}$$

substructured recursively on several levels, where the coupling matrix is incorporated into the substructuring process. Hence, any substructure may consist solely of degrees of freedom from the fluid or from the solid, or caused by the coupling matrix it may contain degrees of freedom of either type.

The crucial point is to modify the AMLS algorithm such that the structure is preserved for the reduced problem, and all eigenvalues are still real.

Pure fluid or solid substructures obviously can be treated in the same way as in the symmetric AMLS method, i.e. they can be decoupled in the stiffness matrix by Gaussian elimination and reduced by modal truncation. We now describe a typical general reduction step.

Structure preserving AMLS

After a couple of reduction steps one arrives at the following matrices where the unknowns have been reordered appropriately

$$\begin{pmatrix} K_{pp} & 0 & K^f_{p\ell} & 0 & K^f_{pi} \\ K^s_{\ell p} & K^s_{\ell\ell} & C_\ell & K^s_{\ell i} & C_{\ell i} \\ 0 & 0 & K^f_{\ell\ell} & 0 & K^f_{\ell i} \\ K^s_{ip} & K^s_{i\ell} & C_{i\ell} & K^s_{ii} & C_i \\ 0 & 0 & K^f_{i\ell} & 0 & K^f_{ii} \end{pmatrix}, \begin{pmatrix} K_{pp} & M^s_{p\ell} & M^f_{p\ell} & M^s_{pi} & M^f_{pi} \\ M^s_{\ell p} & M^s_{\ell\ell} & 0 & M^s_{\ell i} & 0 \\ M^f_{\ell p} & -C^T_\ell & M^f_{\ell\ell} & -C^T_{i\ell} & M^f_{\ell i} \\ M^s_{ip} & M^s_{i\ell} & 0 & M^s_{ii} & 0 \\ M^f_{ip} & -C^T_{\ell i} & M^f_{i\ell} & -C^T_i & M^f_{ii} \end{pmatrix}. \qquad (1)$$

Here p denotes the degrees of freedom obtained in the reduction steps on previous levels, ℓ collects the degrees of freedom to be handled in the current step, i corresponds to the index set of parent substructures.

Structure preserving AMLS

After a couple of reduction steps one arrives at the following matrices where the unknowns have been reordered appropriately

$$\begin{pmatrix} K_{pp} & 0 & K^f_{p\ell} & 0 & K^f_{pi} \\ K^s_{\ell p} & K^s_{\ell\ell} & C_\ell & K^s_{\ell i} & C_{\ell i} \\ 0 & 0 & K^f_{\ell\ell} & 0 & K^f_{\ell i} \\ K^s_{ip} & K^s_{i\ell} & C_{i\ell} & K^s_{ii} & C_i \\ 0 & 0 & K^f_{i\ell} & 0 & K^f_{ii} \end{pmatrix}, \begin{pmatrix} K_{pp} & M^s_{p\ell} & M^f_{p\ell} & M^s_{pi} & M^f_{pi} \\ M^s_{\ell p} & M^s_{\ell\ell} & 0 & M^s_{\ell i} & 0 \\ M^f_{\ell p} & -C^T_\ell & M^f_{\ell\ell} & -C^T_{i\ell} & M^f_{\ell i} \\ M^s_{ip} & M^s_{i\ell} & 0 & M^s_{ii} & 0 \\ M^f_{ip} & -C^T_{\ell i} & M^f_{i\ell} & -C^T_i & M^f_{ii} \end{pmatrix}. \qquad (1)$$

Here p denotes the degrees of freedom obtained in the reduction steps on previous levels, ℓ collects the degrees of freedom to be handled in the current step, i corresponds to the index set of parent substructures.

Next one annihilates $K^s_{\ell i}$ and $K^f_{\ell i}$ by symmetric block Gaussian elimination, and ...

Structure preserving AMLS

solving the substructure eigenvalue problem

$$\begin{pmatrix} K^s_{\ell\ell} & C_\ell \\ 0 & K^f_{\ell\ell} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} = \omega \begin{pmatrix} M^s_{\ell\ell} & 0 \\ -C^T_\ell & M^f_{\ell\ell} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix}.$$

Structure preserving AMLS

solving the substructure eigenvalue problem

$$\begin{pmatrix} K^s_{\ell\ell} & C_\ell \\ 0 & K^f_{\ell\ell} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} = \omega \begin{pmatrix} M^s_{\ell\ell} & 0 \\ -C^T_\ell & M^f_{\ell\ell} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix}.$$

and normalizing the eigenvectors such that

$$\left(\phi_i^T, \frac{1}{\omega_i}\psi_i^T\right) \begin{pmatrix} M^s_{\ell\ell} & 0 \\ -C^T_\ell & M^f_{\ell\ell} \end{pmatrix} \begin{pmatrix} \phi_j \\ \psi_j \end{pmatrix} = \delta_{ij}$$

Structure preserving AMLS

solving the substructure eigenvalue problem

$$\begin{pmatrix} K_{\ell\ell}^s & C_\ell \\ 0 & K_{\ell\ell}^f \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} = \omega \begin{pmatrix} M_{\ell\ell}^s & 0 \\ -C_\ell^T & M_{\ell\ell}^f \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix}.$$

and normalizing the eigenvectors such that

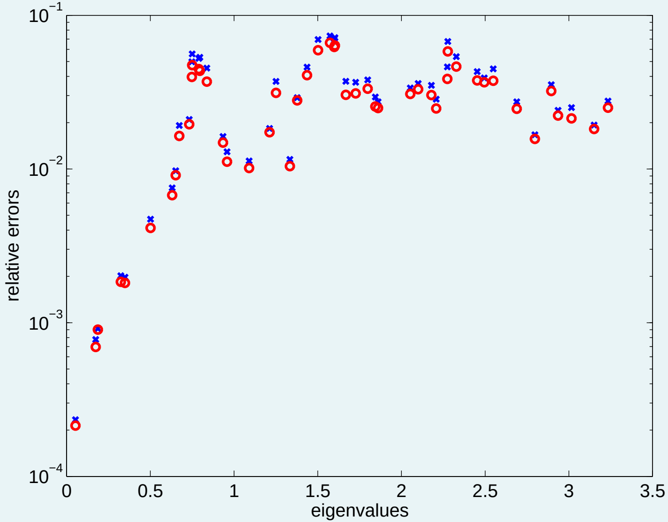
$$\left(\phi_i^T, \frac{1}{\omega_i}\psi_i^T\right) \begin{pmatrix} M_{\ell\ell}^s & 0 \\ -C_\ell^T & M_{\ell\ell}^f \end{pmatrix} \begin{pmatrix} \phi_j \\ \psi_j \end{pmatrix} = \delta_{ij}$$

the oblique projection

$$\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & \Phi_\ell^T & \Omega^{-1}\Psi_\ell^T & 0 \\ 0 & 0 & 0 & I \end{pmatrix} (K_\ell - \lambda M_\ell) \begin{pmatrix} I & 0 & 0 \\ 0 & \Phi_\ell & 0 \\ 0 & \Psi_\ell & 0 \\ 0 & 0 & I \end{pmatrix} = 0$$

preserves the structure of (1).

Structure preserving AMLS



Outline

AMLS for fluid-solid vibrations

Consider the symmetric eigenproblem

$$\left[\begin{pmatrix} 0 & C & K_s & 0 \\ C^T & 0 & 0 & K_f \\ K_s & 0 & 0 & 0 \\ 0 & K_f & 0 & 0 \end{pmatrix} - \lambda \begin{pmatrix} M_s & 0 & 0 & 0 \\ 0 & M_f & 0 & 0 \\ 0 & 0 & K_s & 0 \\ 0 & 0 & 0 & K_f \end{pmatrix}\right] x = 0$$

AMLS for fluid-solid vibrations

Consider the symmetric eigenproblem

$$\left[\begin{pmatrix} 0 & C & K_s & 0 \\ C^T & 0 & 0 & K_f \\ K_s & 0 & 0 & 0 \\ 0 & K_f & 0 & 0 \end{pmatrix} - \lambda \begin{pmatrix} M_s & 0 & 0 & 0 \\ 0 & M_f & 0 & 0 \\ 0 & 0 & K_s & 0 \\ 0 & 0 & 0 & K_f \end{pmatrix}\right] x = 0$$

If $(\lambda^2, (x_s^T, x_f^T)^T)$ is an eigenpair of the fluid-solid-problem then

$$(\pm\lambda, (\lambda^2 x_s^T \quad \pm\lambda x_f^T \quad \pm\lambda x_s^T \quad x_f^T)^T)$$

are solutions of the symmetric problem above unless $\lambda = 0$.

AMLS for fluid-solid vibrations

Consider the symmetric eigenproblem

$$\left[\begin{pmatrix} 0 & C & K_s & 0 \\ C^T & 0 & 0 & K_f \\ K_s & 0 & 0 & 0 \\ 0 & K_f & 0 & 0 \end{pmatrix} - \lambda \begin{pmatrix} M_s & 0 & 0 & 0 \\ 0 & M_f & 0 & 0 \\ 0 & 0 & K_s & 0 \\ 0 & 0 & 0 & K_f \end{pmatrix}\right] x = 0$$

If $(\lambda^2, (x_s^T, x_f^T)^T)$ is an eigenpair of the fluid-solid-problem then

$$(\pm\lambda, (\lambda^2 x_s^T \quad \pm\lambda x_f^T \quad \pm\lambda x_s^T \quad x_f^T)^T)$$

are solutions of the symmetric problem above unless $\lambda = 0$.

Partitioning based on the union of the sparsity structures of the matrices K and M gives an $(s+f)$-dimensional partitioning which can be expanded to an $2(s+f)$-dimensional partitioning so that for $i = 1, \dots, s+f$ the ith and $(i+s+f)$th degree of freedom belong to the same substructure or interface.

AMLS for fluid-solid vibrations

Consider the symmetric eigenproblem

$$\left[\begin{pmatrix} 0 & C & K_s & 0 \\ C^T & 0 & 0 & K_f \\ K_s & 0 & 0 & 0 \\ 0 & K_f & 0 & 0 \end{pmatrix} - \lambda \begin{pmatrix} M_s & 0 & 0 & 0 \\ 0 & M_f & 0 & 0 \\ 0 & 0 & K_s & 0 \\ 0 & 0 & 0 & K_f \end{pmatrix}\right] x = 0$$

If $(\lambda^2, (x_s^T, x_f^T)^T)$ is an eigenpair of the fluid-solid-problem then

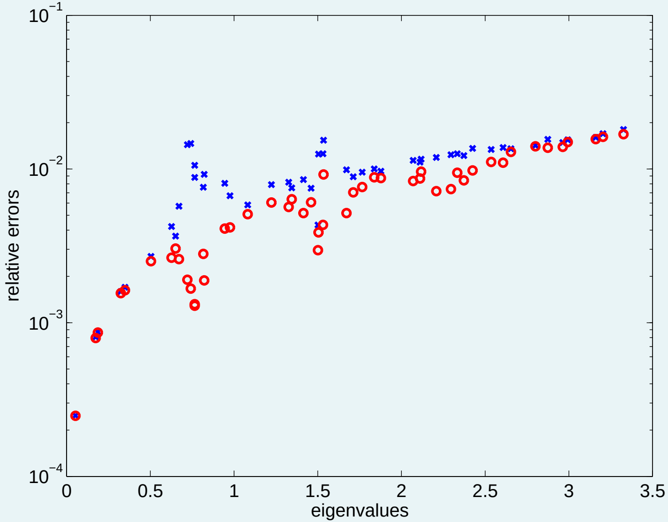
$$(\pm\lambda, (\lambda^2 x_s^T \quad \pm\lambda x_f^T \quad \pm\lambda x_s^T \quad x_f^T)^T)$$

are solutions of the symmetric problem above unless $\lambda = 0$.

Partitioning based on the union of the sparsity structures of the matrices K and M gives an $(s+f)$-dimensional partitioning which can be expanded to an $2(s+f)$-dimensional partitioning so that for $i = 1, \dots, s+f$ the ith and $(i+s+f)$th degree of freedom belong to the same substructure or interface.

Standard AMLS can be performed at nearly the same cost as AMLS for an $(s+f)$ dimensional problem.

AMLS for fluid-solid vibrations



A priori bound for fluid-solid vibrations

CMS method Denote by $0 = \lambda_{+1} < \lambda_{+2} \leq \lambda_{+3} \leq \dots < \omega$ one zero eigenvalue and the positive eigenvalues of the fluid-solid eigenvalue problem and by $0 = \tilde{\lambda}_{+1} < \tilde{\lambda}_{+2} \leq \tilde{\lambda}_{+3} \leq \dots < \omega$ the corresponding eigenvalues of the reduced eigenproblem. Then it holds that

$$\lambda_{+j} - \frac{\lambda_{+j}^2}{\omega + \lambda_{+j}} \leq \tilde{\lambda}_{+j} \leq \lambda_{+j} + \frac{\lambda_{+j}^2}{\omega - \lambda_{+j}}.$$

A priori bound for fluid-solid vibrations

CMS method Denote by $0 = \lambda_{+1} < \lambda_{+2} \le \lambda_{+3} \le \dots < \omega$ one zero eigenvalue and the positive eigenvalues of the fluid-solid eigenvalue problem and by $0 = \tilde{\lambda}_{+1} < \tilde{\lambda}_{+2} \le \tilde{\lambda}_{+3} \le \dots < \omega$ the corresponding eigenvalues of the reduced eigenproblem. Then it holds that

$$\lambda_{+j} - \frac{\lambda_{+j}^2}{\omega + \lambda_{+j}} \le \tilde{\lambda}_{+j} \le \lambda_{+j} + \frac{\lambda_{+j}^2}{\omega - \lambda_{+j}}.$$

AMLS method Consider the AMLS algorithm for fluid-solid interaction eigenproblems on p levels. Denote by $\lambda_{+j}^{(k)}$ the eigenvalues after the k lowest partitioning levels have been handled ($k = 0, \dots, p$) and assume that the cut-off frequency satisfies $\omega > p\lambda_{+j}^{(0)} \ge 0$.
Then the eigenvalues can be bounded by

$$\frac{\omega\lambda_{+j}^{(p)}}{\omega + p\lambda_{+j}^{(p)}} \le \lambda_{+j}^{(0)} \le \frac{\omega\lambda_{+j}^{(p)}}{\omega - p\lambda_{+j}^{(p)}} \tag{2}$$

Conclusions

The unsymmetric eigenvalue problem governing free vibrations of fluid-solid structures has only real eigenvalues.

Conclusions

The unsymmetric eigenvalue problem governing free vibrations of fluid-solid structures has only real eigenvalues.

Taking advantage of a symmetric, positive definite eigenvalue problem which has the same eigenvalues as the original unsymmetric eigenvalue problem, AMLS reasonably takes care of the coupling.

Conclusions

The unsymmetric eigenvalue problem governing free vibrations of fluid-solid structures has only real eigenvalues.

Taking advantage of a symmetric, positive definite eigenvalue problem which has the same eigenvalues as the original unsymmetric eigenvalue problem, AMLS reasonably takes care of the coupling.

The resulting AMLS method requires essentially the same cost as AMLS for a symmetric problem of the original dimension.

Conclusions

The unsymmetric eigenvalue problem governing free vibrations of fluid-solid structures has only real eigenvalues.

Taking advantage of a symmetric, positive definite eigenvalue problem which has the same eigenvalues as the original unsymmetric eigenvalue problem, AMLS reasonably takes care of the coupling.

The resulting AMLS method requires essentially the same cost as AMLS for a symmetric problem of the original dimension.

A priori bounds for CMS and AMLS for the fluid-solid vibration problem can be shown taking advantage of a minmax characterization for nonlinear eigenvalue problems.