Free Vibrations of Fluid-Solid Structures with Strong Coupling

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Outline

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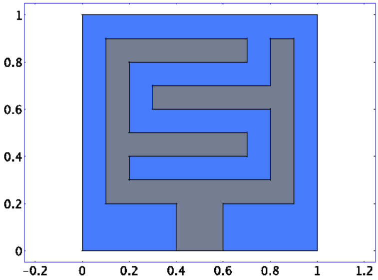
Problem definition

Vibrations of fluid-solid structures can be modelled in terms of solid displacement and fluid pressure and one obtains the classical form of an eigenproblem

$$\begin{aligned} \operatorname{div}\,[\sigma(u)] + \omega^2 \rho_s u &= 0 \text{ in } \Omega_s, \\ \Delta p + \frac{\omega^2}{c^2} p &= 0 \text{ in } \Omega_f, \\ \sigma(u)\cdot n - pn &= 0 \text{ on } \Gamma_I, \\ \nabla p \cdot n + \omega^2 \rho_f u \cdot n &= 0 \text{ on } \Gamma_I, \\ u &= 0 \text{ on } \Gamma_D, \\ \nabla p \cdot n &= 0 \text{ on } \Gamma_N, \end{aligned}$$



where

- u: solid displacement
- p: fluid pressure
- $\lambda = \omega^2$: eigenparameter
- $\sigma(u)$: linearized stress tensor
- ρ_s, ρ_f: densities of solid and fluid

Variational formulation

Find λ and $(u,p) \in H := (H_0^1(\Omega_s))^3 \times H_1(\Omega_f)$, $(u,p) \neq 0$ such that

$$\int\limits_{\Omega_s} \sigma(u) : \varepsilon(v)\,dx + \int\limits_{\Omega_f} \frac{1}{\rho_f} \nabla p \cdot \nabla q\,dx + \int\limits_{\Gamma_I} p\,n \cdot v\,ds$$

$$= \lambda \left(\int\limits_{\Omega_s} \rho_s uv\,dx + \int\limits_{\Omega_f} \frac{1}{\rho_f c^2} pq\,dx - \int\limits_{\Gamma_I} q\,u \cdot n\,ds \right)$$

for every $(v,q) \in H$.

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for every $(v, q) \in H$.

Properties:

- There exists a countable set of eigenvalues the only cluster point of which is ∞
- All eigenvalues are real and non-negative

Discretization

Discretization by finite elements yields an unsymmetric matrix eigenproblem

$$Kx := \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix} =: \lambda Mx, \tag{1}$$

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where

- $K_s, M_s \in \mathbb{R}^{s\times s}$ are symmetric positive definite stiffness and mass matrices of the solid,
- $K_f, M_f \in \mathbb{R}^{f\times f}$ are symmetric stiffness and mass matrices of the fluid, where K_f is positive semidefinite and M_f positive definite,
- $C \in \mathbb{R}^{s\times f}$ is due to the coupling effects between fluid and solid,
- $x_s \in \mathbb{R}^s$ is the solid displacement vector, and
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The computational solution of Problem (1) with standard sparse eigensolvers involves considerable complications. As a remedy some authors prefer an alternative modeling which additionally involves the fluid displacement potential. Then the resulting system is symmetric and efficient methods such as the shift-and-invert Lanczos method applies. As a drawback, however, the dimension of the problem increases considerably.

Common numerical approach

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One first determines the eigenpairs of the symmetric and definite eigenvalue problems

$$K_s x_s = \omega_s M_s x_s \quad \text{and} \quad K_f x_f = \omega_f M_f x_f \tag{2}$$

by the Lanczos method or automated multi-level sub-structuring, and then projects problem (1) to diag$\{X_s, X_f\}$, where the columns of X_s and X_f are the eigenmodes of problem (4) not exceeding a given cut-off level.

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The projected problem

$$\begin{bmatrix} X_s^T K_s X_s & X_s^T C X_f \\ 0 & X_f^T K_f X_f \end{bmatrix} \begin{bmatrix} y_s \\ y_f \end{bmatrix} = \lambda \begin{bmatrix} X_s^T M_s X_s & 0 \\ -X_f^T C^T X_s & X_f^T M_f X_f \end{bmatrix} \begin{bmatrix} y_s \\ y_f \end{bmatrix} \tag{3}$$

has the same structure as the original problem but is of much smaller dimension.

Example

Consider a two dimensional coupled structure consisting of steel and air portions discretized by finite elements. The resulting problem has 120473 degrees of freedom, 67616 of which are located in the solid region and 52857 in the fluid part.

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Table: STEEL-AIR-STRUCTURE

coupled	steel	air	rel.dev. [%]	proj. [%]
0.00		0.00		
41.25		41.32	0.16	2.5e-4
48.67		48.71	0.08	2.7e-4
56.96	56.90		0.11	2.2e-3
75.55	75.51		0.06	3.3e-3
93.18		93.19	0.01	1.0e-4
129.99		130.04	0.05	6.1e-4
150.94	151.03		0.06	3.5e-3
158.16		158.18	0.01	1.8e-4
186.64	186.66		0.12	4.2e-3

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If the fluid air is replaced by water, the scene changes completely.

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Table: STEEL-WATER-STRUCTURE

coupled	steel	water	proj.	rel.error [%]
0.00	56.90	0.00	0.00	
28.01	75.51	178.63	28.33	1.2
41.54	151.03	210.64	43.01	3.5
92.73	186.66	402.93	101.98	10.0
124.70	225.54		133.60	7.1
138.26	451.76		141.87	2.6
270.40	472.45		285.18	5.5
321.79			343.80	6.8
388.73			416.87	7.2
402.77			439.83	9.2

Outline

An inverse-free Rayleigh functional

Important property: If $x := \begin{pmatrix} x_s \\ x_f \end{pmatrix}$ is a right eigenvector of (1) corresponding to the eigenvalue λ, then $\hat{x} := \begin{pmatrix} \lambda x_s \\ x_f \end{pmatrix}$ is a left eigenvector.

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For a given right eigenvector $\begin{pmatrix} x_s \\ x_f \end{pmatrix}$ corresponding to the eigenvalue λ it holds

$$\lambda = \frac{\begin{pmatrix} \lambda x_s \\ x_f \end{pmatrix}^T \begin{pmatrix} K_s & C \\ 0 & K_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix}}{\begin{pmatrix} \lambda x_s \\ x_f \end{pmatrix}^T \begin{pmatrix} M_s & 0 \\ -C^T & M_f \end{pmatrix} \begin{pmatrix} x_s \\ x_f \end{pmatrix}} = \frac{\lambda x_s^T K_s x_s + \lambda x_s^T C x_f + x_f^T K_f x_f}{\lambda x_s^T M_s x_s + x_f^T M_f x_f - x_f^T C^T x_s}.$$

An inverse-free Rayleigh functional cont.

This suggests to define a Rayleigh functional p for some general $s+f$-dimensional vector by the requirement

$$p(x_s, x_f) = \frac{p(x_s, x_f)x_s^T K_s x_s + p(x_s, x_f)x_s^T C x_f + x_f^T K_f x_f}{p(x_s, x_f)x_s^T M_s x_s - x_f^T C^T x_s + x_f^T M_f x_f}$$

which leads to the quadratic equation

$$p(x_s, x_f)^2 x_s^T M_s x_s + p(x_s, x_f)(x_f^T M_f x_f - x_s^T K_s x_s - 2x_s^T C x_f) - x_f^T K_f x_f = 0.$$

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We therefore choose the unique positive root of this equation as Rayleigh functional.

Rayleigh functional

Definition

$$p(x_s, x_f) := \begin{cases} q(x_s, x_f) + \sqrt{q(x_s, x_f)^2 + \frac{x_f^T K_f x_f}{x_s^T M_s x_s}} & \text{if} \quad x_s \neq 0 \\ \frac{x_f^T K_f x_f}{x_f^T M_f x_f} & \text{if} \quad x_s = 0 \end{cases} \tag{4}$$

where

$$q(x_s, x_f) := \frac{x_s^T K_s x_s - x_f^T M_f x_f + 2 x_s^T C x_f}{2 x_s^T M_s x_s}$$

is called Rayleigh functional of the fluid-solid vibration eigenvalue problem (1).

Properties

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Differentiating the defining equation

$$p(x_s, x_f)^2 x_s^T M_s x_s + p(x_s, x_f)(x_f^T M_f x_f - x_s^T K_s x_s - 2x_s^T C x_f) - x_f^T K_f x_f = 0$$

of the Rayleigh functional yields

Lemma

Any right eigenvector x of problem (1) is a stationary point of p, i.e.

$$\nabla p(x) = 0.$$

Variational characterizations

As for symmetric eigenproblems one can derive variational characterizations of eigenvalues.

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Proposition (Rayleigh's principle)

$$\begin{aligned} \lambda_k &= \min\{p(x) : x^T \tilde{M} x_j = 0, j = 1, \dots, k-1\} \\ &= \max\{p(x) : x^T \tilde{M} x_j = 0, j = k+1, \dots, s+f\} \end{aligned}$$

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Proposition (Minmax characterization)

$$\lambda_k = \min_{dim S_k = k} \max_{0 \neq x \in S_k} p(x) = \max_{dim S_k = s+f+1-k} \min_{0 \neq x \in S_k} p(x).$$

Structure preserving projection

Proposition

Assume that $V = \begin{pmatrix} V_s & 0 \\ 0 & V_f \end{pmatrix} \in \mathbb{R}^{s+f \times k}$ *has maximal rank* k.

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and let $\tilde\lambda_1 \le \tilde\lambda_2 \le \cdots \le \tilde\lambda_k$ *be the eigenvalues of the projected problem*

$$K_V y = \tilde\lambda M_V y.$$

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Then it holds that

$$\lambda_j \le \tilde\lambda_j, \quad j = 1, \dots, k.$$

In particular, the jth eigenvalue λ_j does not exceed the jth eigenvalue of the structure eigenproblems $K_s x_s = \lambda M_s x_s$ and of the fluid eigenvalue problem $K_f x_f = \lambda M_f x_f$.

Rayleigh functional iteration

Require: initial vector $x^{(0)}$, $k = 0$

repeat

  evaluate Rayleigh functional $\rho_k := p(x^{(k)})$

  solve $(K - \rho_k M)x^{(k+1)} = Mx^{(k)}$ for $x^{(k+1)}$

  Normalize $x^{(k+1)}$ by $\tilde{M}$

  $k \leftarrow k + 1$

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Similar as for the Rayleigh quotient iteration for symmetric eigenproblems, local cubic convergence can be shown.

Proposition

The iterates ρ_k and $x^{(k)}$ converge locally cubically towards an eigenvalue λ and a corresponding eigenvector x.

Outline

Iterative projection methods

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can be solved only approximately which destroys the fast convergence.

To overcome these drawbacks, one considers iterative projection methods where the projection space V is expanded by an approximation to the direction of the Rayleigh functional iteration.

Jacobi-Davidson method

Expanding the current subspace V_k by $x^{(k+1)} := (K - \rho_k M)^{-1} M x^{(k)}$ is equivalent to expanding it by $t := x^{(k+1)} + \alpha x^{(k)}$ for every $\alpha \in \mathbb{R}$.

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The most robust expansion of this type with respect to inexact solves of $(K - \rho_k M) x^{(k+1)} = M x^{(k)}$ satisfies ${x^{(k)}}^T t = 0$ (cf. V. 2007)

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which leads to the equivalent so called correction equation

$$(I - \frac{Mxx^T}{x^T Mx})(K - \rho_k M)(I - \frac{xx^T}{x^T x})t = -(K - \rho_k M)x, \quad t^T x = 0$$

for a given eigenpair approximation (ρ_k, x).

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Iterative projection methods of this type are known as Jacobi-Davidson method and were introduced by Sleijpen and van der Vorst (1996) and were studied for nonlinear eigenproblems in Betcke, V. (2004), V. (2007).

structure preserving Jacobi-Davidson method

The standard approach of the Jacobi-Davidson method, i.e. considering

$$V^T K V y = \lambda V^T M V y \tag{6}$$

where V is expanded by the solution of the correction equation destroys the structure of problem (1).

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To preserve the structure of the fluid-solid eigenproblems and to ensure real eigenvalues of the projected eigenproblem we expand the ansatz space in every step by 2 vectors $\begin{pmatrix} t_s \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ t_f \end{pmatrix}$ where $t = \begin{pmatrix} t_s \\ t_f \end{pmatrix}$ solves approximately the correction equation. Hence, we use structure preserving projection matrices

$$V = \begin{pmatrix} V_s & 0 \\ 0 & V_f \end{pmatrix}$$

and obtain

$$V^T K V = \begin{pmatrix} V_s^T K_s V_s & V_s^T C V_f \\ 0 & V_f^T K_f V_f \end{pmatrix} \text{ and } V^T M V = \begin{pmatrix} V_s^T M_s V_s & 0 \\ -V_f^T C^T V_s & V_f^T M_f V_f \end{pmatrix}.$$

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Proposition

Let $\lambda_j^{(k)}$ be the j smallest eigenvalue of the kth projected eigenproblem of the structure preserving Jacobi-Davidson method.

Then it holds that

$$\lambda_j \le \lambda_j^{(k+1)} \le \lambda_j^{(k)} \text{ for } k = 1, \dots, \dim(V_k).$$

Structure preserving Jacobi-Davidson method

REQUIRE Initial basis $V = \text{diag}\{V_s, V_f\}$, $V_s^T K_s V_s = I$, $V_f^T M_f V_f = I$,
$m = 1$; θ_m=0;

determine preconditioner $L \approx (K - \sigma M)^{-1}$, $\sigma \approx \lambda_{\min}$

while $\theta_m \leq$ maxeig **do**

solve the projected problem

$$\begin{pmatrix} V_s^T K_s V_s & V_s^T C V_f \\ 0 & V_f^T K_f V_f \end{pmatrix} \begin{pmatrix} y_s \\ y_f \end{pmatrix} = \theta \begin{pmatrix} V_s^T M_s V_s & 0 \\ -V_f^T C^T V_s & V_f^T M_f V_f \end{pmatrix} \begin{pmatrix} y_s \\ y_f \end{pmatrix}$$

choose m smallest eigval. θ_m and corresp. eigvec. $(y_s^T, y_f^T)^T$

determine Ritz vector $x = \begin{pmatrix} V_s y_s \\ V_f y_f \end{pmatrix}$ and residual $r = (K - \theta_m M)x$

if $\|r\|/\|x\| < \epsilon$

while $\|r\|/\|x\| < \epsilon$

accept approximate mth eigenpair (θ_m, x); increase $m \leftarrow m + 1$;

choose m smallest eigval. θ_m and corresp. eigvec. $(y_s^T, y_f^T)^T$

determ. Ritz vec. $x = \begin{pmatrix} V_s y_s \\ V_f y_f \end{pmatrix}$ and res. $r = (K - \theta_m M)x$

endwhile

reduce search space V if indicated
determine new preconditioner $L \approx (K - \theta M)^{-1}$ if necessary
endif
compute approximate solution $t = (t_s^T, t_f^T)^T$ of the correction equation

$$(I - \frac{Mxx^T}{x^T Mx})(K - \rho_m M)(I - \frac{xx^T}{x^T x})t = r, x^T t = 0$$

orthogonalize $v_s = t_s - V_s V_s^T K_s t_s$, $v_f = t_f - V_f V_f^T M_f t_f$
if $\|v_s\|_{K_s} >$ tol expand $V_s \leftarrow [V_s, v_s / \|v_s\|_{K_s}]$
if $\|v_f\|_{M_f} >$ tol expand $V_f \leftarrow [V_f, v_f / \|v_f\|_{M_f}]$
update projected problem
endwhile

Solving correction equation

The correction equation is solved approximately by a few steps of an iterative solver (GMRES or BiCGStab).

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The operator $T(\rho) := K - \rho M$ is restricted to map the subspace $x^\perp$ into itself. Hence, if $L \approx T(\sigma)^{-1}$ is a preconditioner of $T(\sigma)$, then a preconditioner for an iterativ solver of the correction equation should be modified correspondingly to

$$\tilde{L} := (I - \frac{Mxx^T}{x^TMx})L(I - \frac{xx^T}{x^Tx}).$$

Solving correction equation

The correction equation is solved approximately by a few steps of an iterative solver (GMRES or BiCGStab).

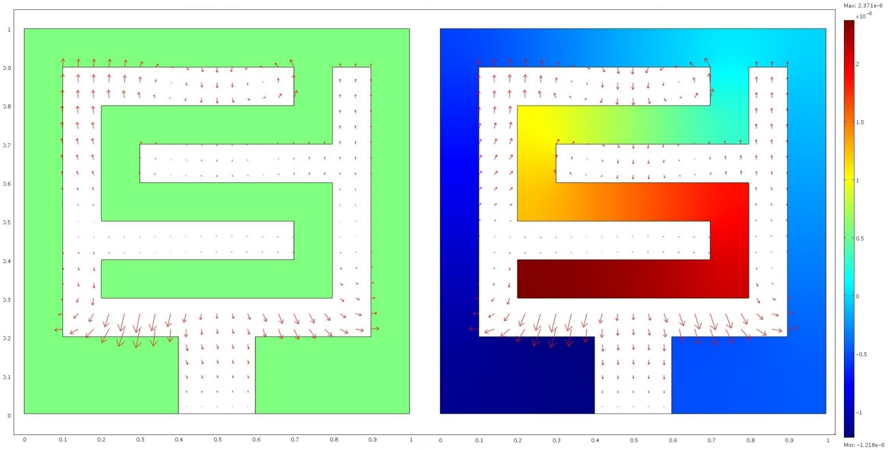
The operator $T(\rho) := K - \rho M$ is restricted to map the subspace $x^\perp$ into itself. Hence, if $L \approx T(\sigma)^{-1}$ is a preconditioner of $T(\sigma)$, then a preconditioner for an iterativ solver of the correction equation should be modified correspondingly to

$$\tilde{L} := (I - \frac{Mxx^T}{x^T Mx})L(I - \frac{xx^T}{x^T x}).$$

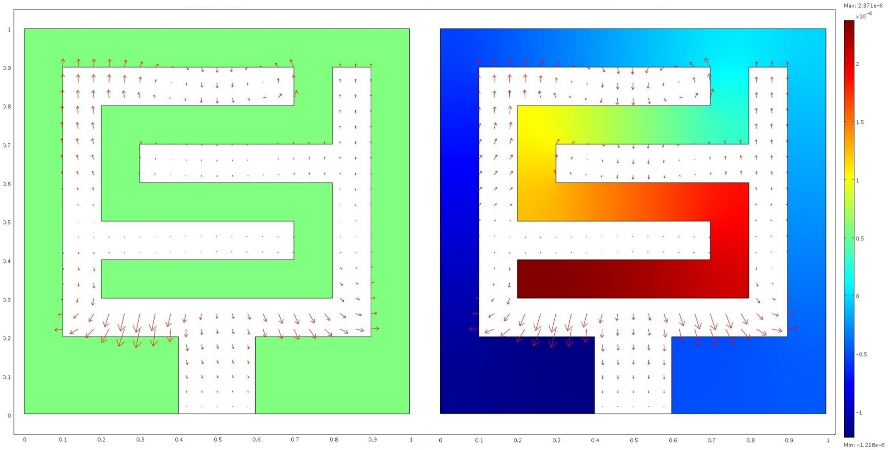
Taking into account the projectors in the preconditioner, i.e. using $\tilde{L}$ instead of L, raises the cost of the preconditioned Krylov solver only slightly (cf. Sleijpen, van der Vorst). Only one additional linear solve with system matrix L is required.

Outline

Numerical Example

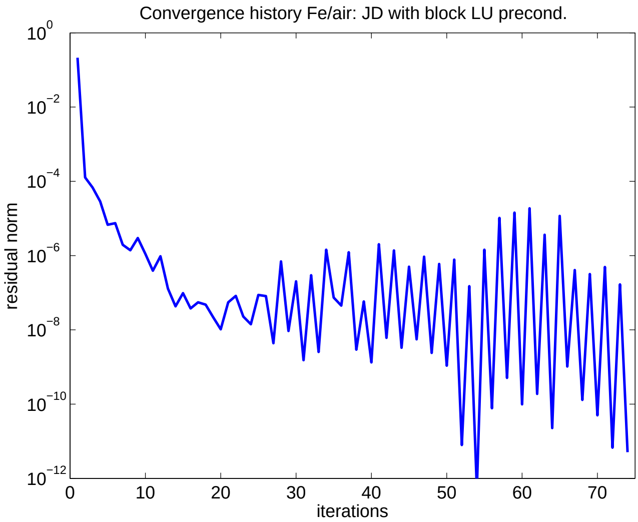


Numerical Example



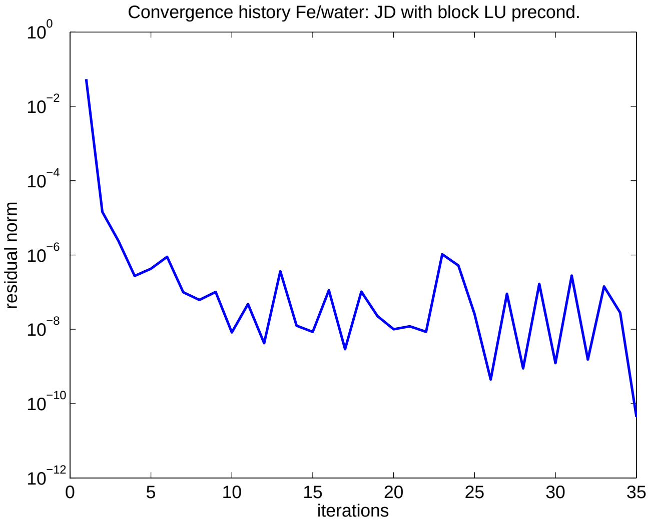
Consider a finite element model of a coupled steel/air system with 120473 DoFs (67616 Fe, 52857 air).
Determine all 23 eigenfrequencies less than 500 Hz.

Numerical Example; Fe-air



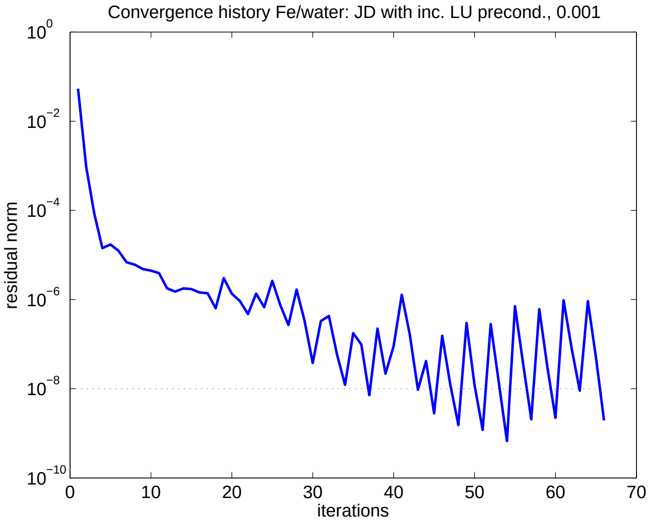
CPU time : 77.9 sec.

Numerical Example; Fe-H_2O



CPU time : 30.0 sec.

Numerical Example; Fe-H_2O



CPU time : 60.5 sec.

Outline

Conclusions

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The Automated Multi-Level Substructuring method (AMLS) can be generalized to fluid-solid eigenproblems (Stammberger, V. (2010)).