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3-D FINITE ELEMENT MODELLING OF GRANULAR FLOW IN SILOS

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ABSTRACT

A continuum approach based on the Finite Element Method is presented in this paper with which the behaviour of granular material during at rest conditions and during flow can be studied. Various numerical and physical aspects of this model are discussed. A 3-dimensional FE-simulation of filling and discharging processes of a rectangular silos with eccentric outlet is presented. The calculated wall loads due to bulk materials are compared with the relevant values from various design Codes.

Keywords: silo, 3d-modelling, interface elements, eccentric outlets, Finite Element Method

1 INTRODUCTION

Worldwide the estimation of the relevant pressures on a silo structure due to bulk materials is still based on the well-known analytical model of Janssen (H. A. Janssen, 1895). This slice model (fig. 1) can only be used for bins with symmetric cross-sections and material at rest. In practice a lot of silos are built with eccentric outlets and inlets respectively or non-circular cross-sections. Next the relevant non-symmetric dynamic discharge pressure, which can be several times greater than the filling one, can not be estimated with this simple model. To overcome this problem the wall loads due to Janssen are increased by more or less empirical overpressure coefficients. This practical approach is unsatisfactory and can lead to an uneconomical or in some cases to an unsafe design, as the high failure rate in silos demonstrates. As an example figure 2 shows the filling and discharge wall pressures for a simple silo with rectangular cross section and centric outlet according to the European 1 Part 4 (DIN ENV 1991-4: 1996) and the American (ACI, 1997) Standard.

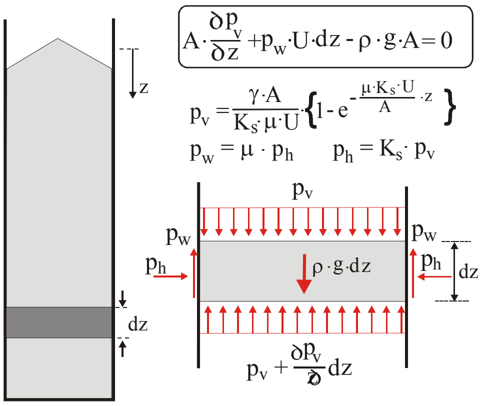

FIG. 1. Model of Janssen

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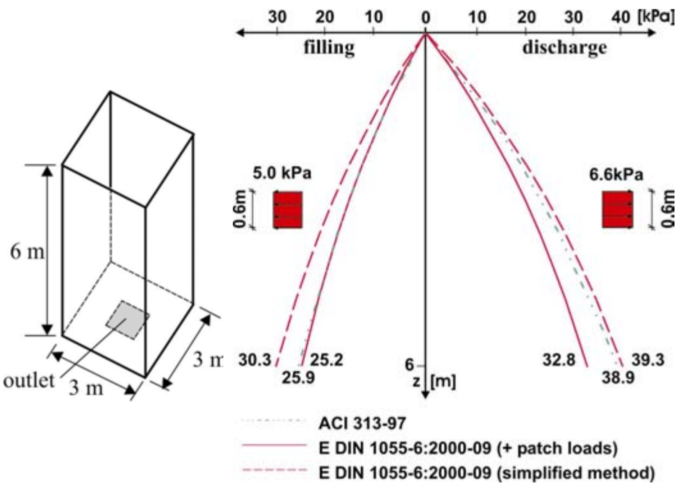
Significant differences can be seen. Eurocode gives 17 % (filling) and 24 % (discharge) higher pressures than the American Code. In both standards the discharge loads p_{he} are estimated by increasing the filling values with a overpressure factor. ACI 313-97 recommend to use a factor between 1.35 and 1.5 whereas EC gives a value of 1.6. According to both Codes the horizontal pressure is symmetric over the perimeter of the bin. ACI gives no information about loads in eccentric discharged bins. Some methods are mentioned in the ACI Committee Report, but they are not recommended. Eccentric outlets and eccentric filling are considered in EC as shown in fig. 3. The given formulae for design are limited to an eccentricity of the outlet of less than $0.25 \cdot d_c$. According to EC 1 Part 4 the filling load p_f is composed of a fixed load according to Janssen and a free load, called patch load p_p (see fig. 2, 4). The latter value depends on the eccentricities of the inlet and outlet. The patch load causes bending moments in the walls The discharge loads are estimated by magnifying the filling pressure by a factor C_0 which depends on the material properties only. For silos, where the hydraulic radius d_c (fig. 3) is less than 5 m the patch load can be omitted when the filling pressure is further increased.



parameters:

bulk material	wheat
weight density γ	$8.5 kN/m^3$
internal angle of friction	33^o
wall friction coefficient μ_m	0.25
eccentricity of outlet	0.0 m
horizontal/vertical pressure ratio K_{sm}	0.50/0.46

FIG. 2. Wall pressures (design values)

From the example mentioned before it can be clearly seen that more accurate methods to determine the relevant loads due to bulk materials are urgently needed. Numerical simulations seems to be an appropriate method to get a better understanding of granular flow and the relevant phenomenae. Experiments are very time consuming, difficult to conduct and only the reaction and not the behaviour of the bulk material within the bin can be studied.

Finite Element simulations of granular flow in silos based on a continuum approach are becoming more and more popular due to the increasing computer power and the capacity of the available program in the last decades. Nevertheless it must be recognized, that the simulation of granular flow in silos is still a demanding task. Some aspects will be discussed in the following.

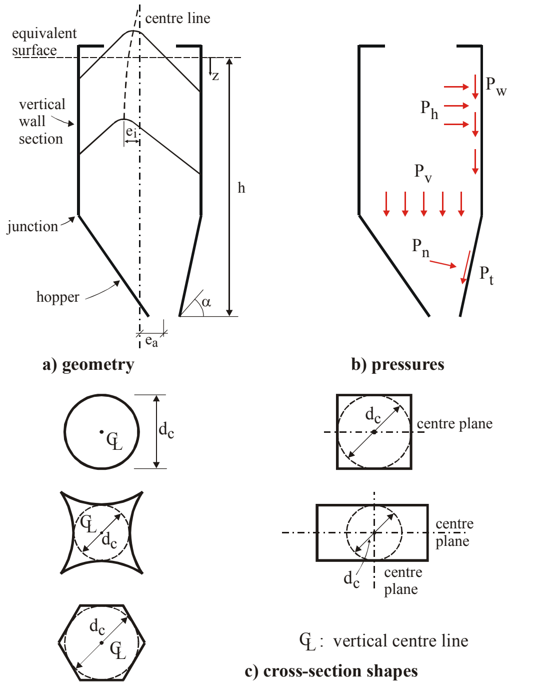


FIG. 3. Silo forms showing dimensions and pressure notation according to EC

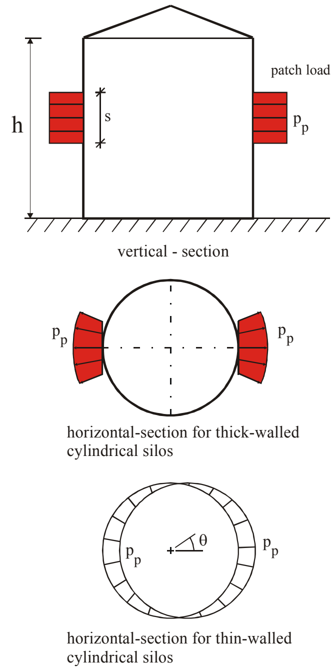


FIG. 4. Side elevation an plan view of the patch load according EC

2 NUMERICAL MODEL

The following important aspects have to be considered when using a Finite Element Program to simulate granular flow:

- *Material model*
  It is obvious that the reliability of any FE-analysis depends on the assumptions and simplifications of the numerical and mechanical model. Here the constitutive model for the bulk material is of greatest importance. It should be out of discussion the simple limit state models like e.g. the Mohr Coulomb failure criteria can certainly not model the complex behaviour of granular bulk materials. Constitutive models as published by Lade (elastic-plastic), Kolymbas (hypoplastic) or v. Wolffersdorff (hypoplastic) for example have to be used in conjunction with a dynamic, viscous part (Weidner, 1990).
- *Filling procedure*
  The silo has to be filled numerically layer by layer. This is of great importance in case of bins with inclined walls (Rombach & Keiter , 2002).
- *Dynamic analysis of granular flow*
  Usually the greatest pressures in silos don't occur during at rest conditions but during emptying of the bin. Therefore one must model the discharge phase. This requires a dynamic mechanical model and code.

- *Boundary conditions*
  The interaction between the granular media and the silo structure has to be modeled. Interface elements and accurate material models (e.g. friction) are required.
- *Numerical algorithms*
  The numerical algorithms must be fast and robust. Correct convergence criterias must be defined.

To take into account all these specific requirements, a special non-linear Finite Element Program 'SILO' based on a continuum approach has been developed to simulate the behavior of non-cohesive granular material in silos during at rest conditions or during flow. Details and background information as well as results for 2-dimensional and axisymmetric conditions can be found in several publications (e.g. Rombach, 1991 and Rombach & Eibl, 1995). This FE-program has been improved in the last years to 3 dimensions to model un-symmetric conditions, like e.g. an eccentric discharged bin. First a 20-node-isoparametric volume element (bulk element) was implemented (fig. 5).

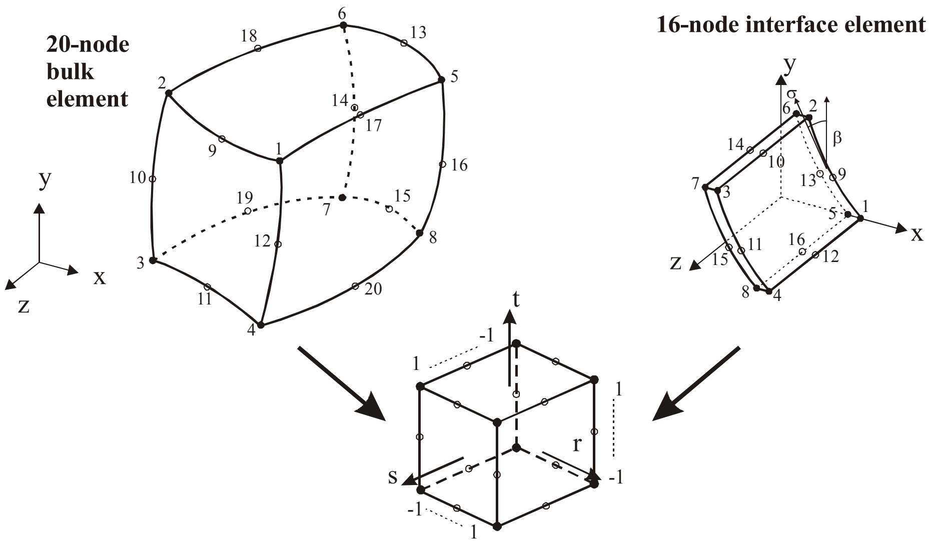


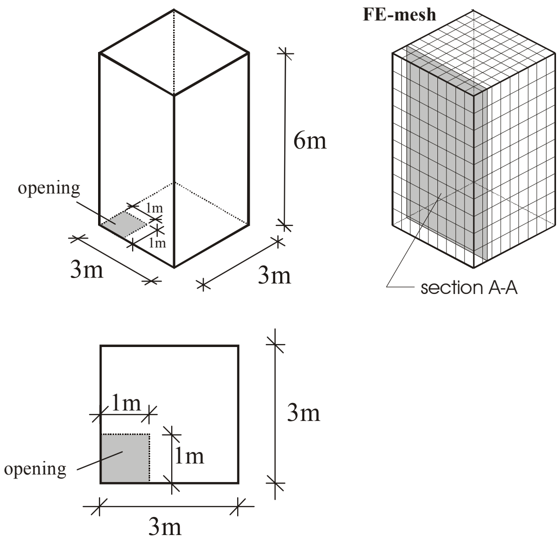
FIG. 5. 20-node-bulk and 16-node interface element

The stresses in the granular media stored in bins are significantly dependant on the interaction between the bulk material and the silo walls. Several interface algorithmens like e.g. point to surface elements, spring elements, contact elements had been studied. The often used point to surface elements have caused great numerical problems. It should be noted that the bulk material is always in contact with the silo walls. Thus complicated contact algorithms are not required. The best results were obtained with a 16-node-Interfaceelement (fig. 5). The shape functions are linear in thickness direction as 4 nodes are omitted. Several comparison analysis had been conducted with different linear and non-linear contact models. Fortunately a linear friction model (Mohr-Coulomb) is sufficient enough in most relevant cases.

Simulation of the discharge processes are extremely time consuming. The size of the element mesh and the analysed discharge time are mostly limited by the time for the computation. Most effort is required for solving the linearized global equilibrium equations. Various solvers based on the Gauss algorithms or iterative procedures had been tested. The best results were obtained with a modified Gauss elimination procedure according to 'Crout' and the 'Umfpack' numerical package. The global stiffness matrix is non-symmetric.

3 3D-ANALYSIS OF AN ECCENTRIC DISCHARGED SILO

In the following the results of a numerical analysis of a rectangular silo with eccentric outlet (geometry and parameters see figure 6) will be discussed. The material model of Kolymbas (Kolymbas, 1988) with the parameters for wheat are used. Very small time steps of 1/1000 s as recommend by Ruckenbrod (Ruckenbrod, 1995) had been used for the following analysis to guarantee convergence.



parameters	
constitutive material model	Kolymbas
bulk material	wheat
density	8.5 kN/m³
internal friction angle	33°
wall friction coefficient	0.25
outlet eccentricity	e_a/d_c = 1/3=0.33
opening time	0.001·200 = 0.2 s

FIG. 6. Silo geometry and section A-A

Results from the 3d-simulation are shown in following figures. The principal stresses of all 2 x 2 integration points per element in section A-A are plotted in fig. 7. During storage the wall pressure and the vertical pressure increases more or less linear with the depth of the bin. A stress arch develops when the restraints of the opening are released. In figure 8 the velocity field of an internal flow can be seen. The discharge rate increases with time.

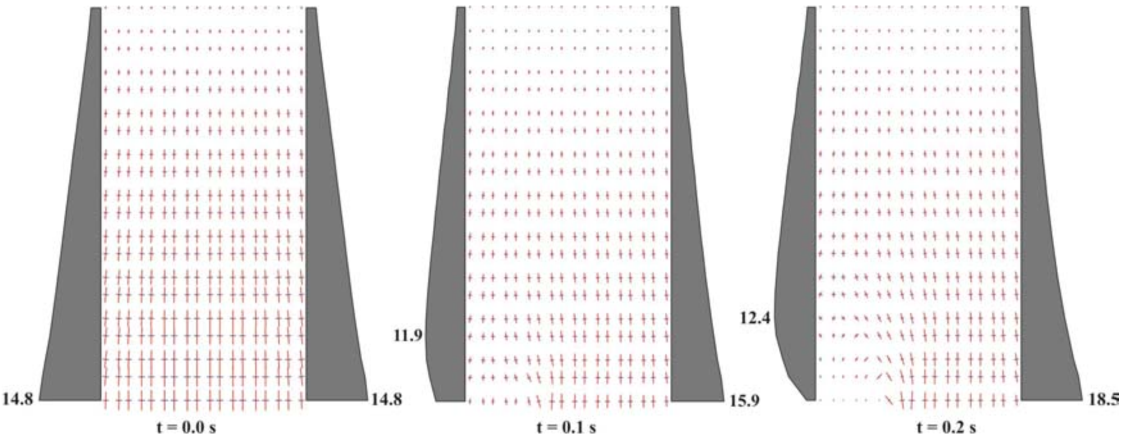


FIG. 7. Wall pressures and principal stresses in different time steps

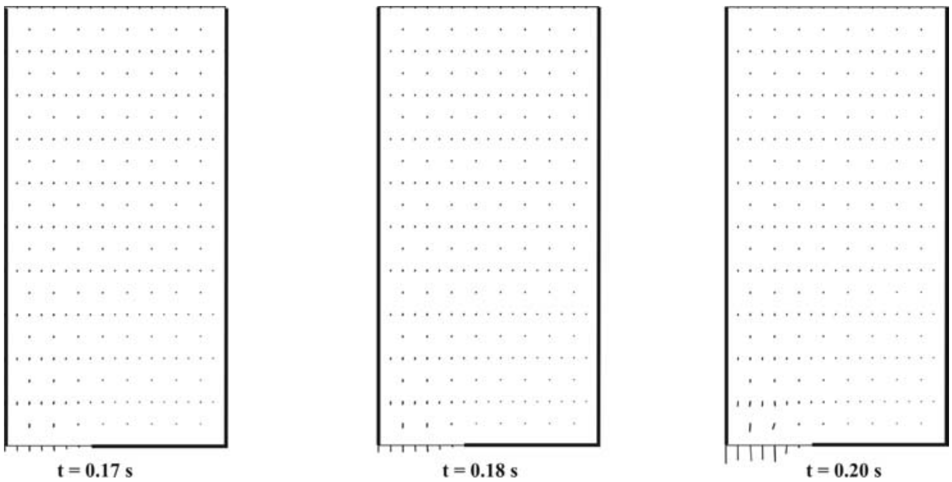


FIG. 8. Velocities in different time steps

Fig. 9 shows the numerical results and the design load according to the European and American Standard. It must be noted here, that the eccentricity of the outlet (≈ 0.33 $\cdot d_c$) is outside of the limit 0.25$\cdot d_c$ given in the Code. Great difference between the numerical and analytical wall loads can be seen. The EC values are on the safe side and seem to be very uneconomical.

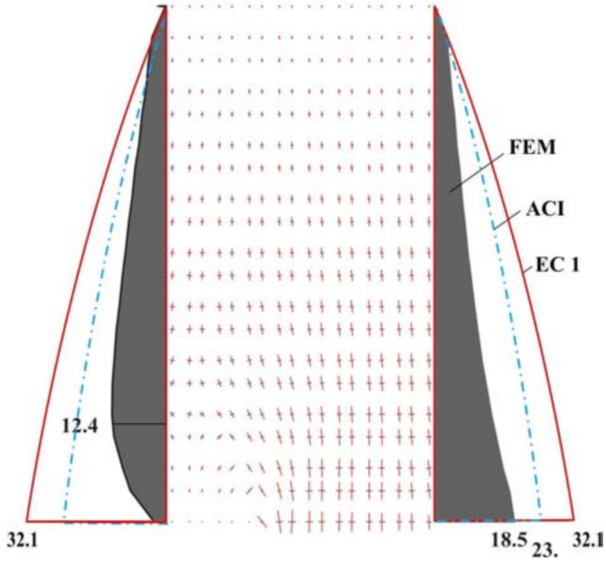


FIG. 9. Horizontal wall pressure (characteristic values)

4 SUMMARY AND OUTLOOK

Various aspects of Finite Element simulations of granular flow in silos were discussed. The stress distribution and the wall pressures of a rectangular bin filled with wheat are presented. Great differences between the relevant horizontal wall load due to bulk materials and the corresponding values from the various Codes are observed which demonstrate the lack in knowledge with regards to silo loads.

The main aim of the ongoing research is to get a better understanding of the relevant phenomenae within the bulk material during flow and to develop more accurate design models.

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